RY-003-001617 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March - 2019

Mathematics : Paper - 602 (A)

(Mathematical Analysis & Abstract Algebra)

Faculty Code : 003
Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

1 Answer the following : **20**

(1) Define Compact Set

(2) Define Least Upper Bound

(3) Define Sequential Compactness

(4) Find $L\left(\cos^2 4t\right)$

(5) Find $L\left(t^n\right)$

(6) Find $L^{-1}\left(\frac{1}{(s-a)^2+b^2}\right)$

(7) Find $L^{-1}\left(\frac{1}{4s+5}\right)$

(8) Find $L^{-1}\left(\frac{3s+4}{s^2+16}\right)$

(9) Define Natural Mapping

(10) $\phi:(G,*)\to(G',\Delta)\phi(x)=x$ Then, show that ϕ is a homomorphism.

(11) Define Kernel of homomorphism

(12) Define Division Ring

(13) Obtain radicals of the rings $\left(Z_{12},+_{12},\times_{12}\right)$

(14) Define Field

(15) Define Left Ideal

(16) Define Equality of polynomials

(17) Define Degree of polynomials

(18) Define Leading coefficient

(19) Define irreducible polynomials

(20) Define Constant Polynomials

2 (A) Answer any **three** out of six : **6**

(1) Show that the sets $A=[1, 2]$ and $B=(2, 3)$ are not separated sets of metric space R

(2) Determine subset $(0, 1)$ of metric space R is open, closed, connected or compact

(3) Let E be a non – empty closed subset of metric space R. If E is lower bounded set then glb E lies in E.

(4) Find $L\left(\cosh^3 2t\right)$

(5) Find $L\left(\sqrt{te^{2t}}\right)$

(6) Find $L^{-1}\left(\dfrac{s}{\left(s^2-1\right)^2}\right)$

(B) Answer any **three** out of six : **9**

(1) Prove that every singleton subset of any metric space is connected set

(2) If $E_n=[-n, n]\ n\in N$ then the collection $\{E_n / n\in N\}$ is a cover of R or Not

(3) Show that the set of all even number is a countable set

(4) State and prove First Shifting theorem of Laplace transformation

(5) Find Laplace transformation of

$f(t)\ f(t)=\begin{Bmatrix} t, 0<t<4 \\ 5, t>4 \end{Bmatrix}$

(6) Find $L^{-1}\left(\dfrac{2s^2-4}{(s+1)(s-2)(s-3)}\right)$

(C) Answer any **two** out of five : **10**

(1) Prove that every open interval of metric space R is an open set

(2) State and prove theorem of Nested intervals

(3) Prove or disprove that arbitrary union of compact sets is compact

(4) Using convolution theorem find $L^{-1}\left(\frac{s}{\left(s^2+4\right)^2}\right)$

(5) Using convolution theorem find

$$L^{-1}\left(\frac{s^2}{\left(s^2+a^2\right)\left(s^2+b^2\right)}\right)$$

3 (A) Answer any **three** out of six : **6**

(1) $\phi:(G,*)\to(G',\Delta)$ be a homomorphism then, if $H\le G$ then $\phi(H)\le G'$

(2) Let I be an ideal of a ring with unity R. Then, $I=R$ if $1\in I$

(3) If $f=(2, 0, -3, 0, 4, 0.....)$ and $g=(1, -2, 0, 0...)$ Are polynomials of $R[X]$ then find $f+g$

(4) State and prove Factor theorem

(5) $U_1=\{f\in C[0,1]/f(0)=0\}$ is subring of $(C[0, 1],+,*)$

(6) $f(x),g(x)\in Z_5[x]$ Where $f(x)=2x^3+4x^2+3x+2$ $g(x)=3x^4+2x+4$ Find $f(x).g(x)$

(B) Answer any **three** out of six : **9**

(1) $\phi:(G,*)\to(G',\Delta)$ be a homomorphism then, if N is normal subgroup o f G then, $\phi(N)$ is a normal subgroup of $\phi(G)$

(2) A homomorphism is $\phi:(G,*)\to(G',\Delta)$ one - one iff $K_\phi=\{e\}$

(3) Is $R = \left\{ a + b\sqrt{2} \,/\, a, b \in Z \right\}$ a ring with respect to the usual addition & multiplication?

(4) Prove that field has no proper Ideal

(5) State and prove Reminder theorem

(6) Express $f(x)$ as $q(x)g(x) + r(x)$ form by using division algorithm for given $f(x)$ & $g(x)$

$$f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$$

$$g(x) = x^2 - 2x + 3 \in Z_5[x]$$

(C) Answer any **two** out of five : **10**

(1) Let $\phi : (G, *) \to (G', \Delta)$ be a homomorphism. Then K_ϕ is a normal subgroup of G.

(2) State and prove First Fundamental theorem of Homomorphism

(3) A commutative ring R with unity is a field if it has no proper Ideal.

(4) Factorize $f(x) = x^4 + 4 \in Z_5[x]$ by using factor theorem

(5) Find g. c. d. of $f(x) = 6x^3 + 5x^2 - 2x + 25$ and $g(x) = 2x^2 - 3x + 5 \in R[X]$ and express it in the form $a(x)f(x) + b(x)g(x)$